Microwave Conductivity of d-Wave Superconductors

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We calculate the microwave conductivity of a $d_{x^2-y^2}$ superconductor in the presence of elastic impurity scattering and inelastic scattering due to antiferromagnetic spin fluctuations. The low-temperature conductivity does not simply reflect the linear temperature dependence of the number of quasiparticles in a d-wave system, as often supposed. We compare with recent data on high-quality $YBa_2Cu_3O_{6.95}$ single crystals.

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Microwave surface impedance measurements of the cuprate superconductors provide information on the real and imaginary parts of the long wavelength ($q \to 0$) conductivity. The imaginary part of the conductivity is proportional to the square of the inverse penetration depth, and its low frequency limit determines the superfluid density. Recent experiments [1] on high purity crystals of $YBa_2Cu_3O_{6.95}$ have found that the superfluid density n_s appears to decrease linearly with T at low temperatures, in contrast to measurements on samples of apparently poorer quality which exhibit a T^2 variation at the lowest temperatures [2]. The linear-T variation of n_s has been interpreted in terms of an unconventional superconducting state with lines of order parameter nodes on the Fermi surface [1,3].

Several authors [4–6] have attempted to account for the measured T^2 dependence of the penetration depth in other samples by postulating the existence of resonant potential scatterers acting as strong pair breakers in the unconventional state. While this model appears to describe several aspects of the penetration depth experiments consistently, it is natural to ask whether a similar approach can be used to understand the frequency and temperature dependence of the real part of the conductivity $\sigma(T,\Omega)$. In this Letter, we examine the consequences of the joint assumptions of d-wave pairing and strong impurity scattering for the conductivity, and compare our results with the experimental data of Bonn *et al.* [7–9].

Conductivity and phenomenological model.— Bonn *et al.* [7,8] use measurements of the surface resistance $R_s = (8\pi^2\Omega^2\lambda^3\sigma)/c^4$ at microwave frequency Ω, together with the penetration depth λ, to determine the conductivity σ. These authors then propose that the real part of the conductivity can be represented by a Drude-like form associated with the excited quasiparticles,

$$\sigma(T,\Omega) = \frac{ne^2}{m} n_{qp}(T) \frac{\tau(T)}{1+\Omega^2\tau^2(T)}, \quad (1)$$

where n is the electron density, $n_{qp}(T) = 1 - n_s(T)$ is the relative number of quasiparticles excited at temperature T, and $\tau(T)$ is the relaxation time in the superconducting state. Bonn *et al.* find that in high purity single crystals, there is a large increase in $\tau(T)$ below T_c, interpreted in terms of a collapse of the inelastic scattering due to the decrease in the spin fluctuation spectral weight as the superconducting gap opens [10]. The increase continues down to a temperature T_0 of order $0.4T_c$, where $\tau(T)$, which has increased by almost 10^2, reaches an impurity-dominated limit where τ appears to saturate at a constant value. Associated with the initial increase in $\tau(T)$, the conductivity σ rises just below T_c, reaching a maximum near T_0 which for $\Omega\tau \ll 1$ can be as large as 10 to 20 times its value at T_c in the best crystals reported. As T falls below T_0, σ decreases, and is observed to follow a linear $-T$ behavior at the lowest temperatures measured, at least for the 34.8 GHz data. This may be understood in terms of Eq. (2) if $n_{qp}(T) \sim T/T_c$, as expected for a d-wave superconductor, and $\tau(T)$ is constant (as discussed below, the latter assumption does not appear to be justified by the microscopic theory). The microwave frequency plays a role when $\Omega\tau \simeq 1$, consistent with Eq. (1), leading to a reduction in the conductivity peak at higher values of Ω, as noted by Bonn *et al.* [7,8].

This framework provides a qualitative as well as a surprisingly quantitative description of the data. However, it is not obvious that a result similar to Eq. (1) obtains in the superconducting state. Furthermore, extraction of the $T \ll T_c$ temperature dependence of the conductivity is difficult because of the problem of residual surface resistance. Typically, one subtracts a term so that the intrinsic surface resistance vanishes at $T=0$, leading to a conductivity $\sigma(T,\Omega)$ which also vanishes. While this is appropriate for an s-wave gap, where σ vanishes as $\exp(-\Delta/T)$, for a d-wave gap one expects [11] that σ approaches a finite limit of order $ne^2/m\Delta_0(0)$, where

$\Delta_0(T)$ is the maximum of the order parameter over the Fermi surface. Here we examine the low temperature and frequency dependence of $\sigma(T,\Omega)$ for such a state. For concreteness and simplicity, we have chosen to work with a $d_{x^2-y^2}$ state, with order parameter $\Delta k=\Delta_0(T)\times\cos 2\phi$, and a cylindrical Fermi surface. However, our conclusions apply qualitatively to all unconventional pairing states with lines of nodes (point nodes in 2D).

Microscopic model.— We adopt as a theoretical framework to discuss the low temperature microwave conductivity a generalized BCS model with an unconventional ($d_{x^2-y^2}$) superconducting state and a self-consistent t-matrix approximation for the one-electron self-energy due to impurity scattering. The surface impedance [12] and conductivity [13–15] in such a model have been calculated for various anisotropic unconventional states, primarily in the context of heavy fermion superconductivity.

The conductivity $\sigma(T,\Omega)$ of such a system was shown in Ref. [13] to be determined by the imaginary part of the bare current response. Vertex corrections due to s-wave impurity scattering vanish identically for singlet states at $q=0$, while those corresponding to order parameter collective modes will be negligible at low frequencies, and in any case vanish for the one-dimensional state considered here. The final result of interest is given by [13]

$$\sigma_{ij}(\Omega)=-\frac{ne^2}{2m\Omega}\int_{-\infty}^{\infty}d\omega\{\tanh[\tfrac{1}{2}\beta\omega]-\tanh[\tfrac{1}{2}\beta(\omega-\Omega)]\}S_{ij}(\omega,\Omega)\,, \tag{2}$$

where

$$S_{ij}(\omega,\Omega)=\mathrm{Im}\int\frac{d\phi}{2\pi}\hat{k}_i\hat{k}_j\left[\frac{\tilde{\omega}'_+}{\tilde{\omega}_+-\tilde{\omega}'_+}\left(\frac{1}{\xi'_{0+}}-\frac{1}{\xi_{0+}}\right)+\frac{\tilde{\omega}'_-}{\tilde{\omega}_+-\tilde{\omega}'_-}\left(\frac{1}{\xi_{0+}}+\frac{1}{\xi'_{0-}}\right)\right]. \tag{3}$$

Here we have put $\tilde{\omega}_\pm=\tilde{\omega}(\omega\pm i0)$ and $\tilde{\omega}'_\pm=\tilde{\omega}(\omega-\Omega\pm i0)$, where $\omega-\tilde{\omega}=\Sigma_0$ is the averaged self-energy due to impurity scattering in a t-matrix approximation, $\Sigma_0=\Gamma G_0/(c^2-G_0^2)$. The parameter $\Gamma\equiv n_i n/\pi N_0$ is a scattering rate depending only on the concentration of defects n_i, the electron density n, and the density of states at the Fermi level, N_0, while the strength of an individual scattering event is characterized by the cotangent of the scattering phase shift, c. The integrated diagonal Green's function averaged over disorder, $G_0\equiv -i\langle\tilde{\omega}/(\tilde{\omega}^2-\Delta_k)^{5/2}\rangle$, depends on the renormalized frequency $\tilde{\omega}$ and must therefore be determined self-consistently. Finally, we note that $\xi_{0\pm}=\mathrm{sgn}(\omega)[\tilde{\omega}_\pm^2-\Delta_k^2]^{1/2}$ and $\xi'_{0\pm}=\mathrm{sgn}(\omega-\Omega)[(\tilde{\omega}'_\pm)^2-\Delta_k^2]^{1/2}$ are chosen to have branch cuts such that $\mathrm{Im}\xi_{0+}>0$, $\mathrm{Im}\xi'_{0+}>0$, and $\mathrm{Im}\xi'_{0-}<0$. The conductivity [(2) and (3)] reduces in the normal state limit $\Delta_k\to 0$ to the Drude form, $\sigma_{ij}=\delta_{ij}\sigma_0/[1+(\Omega\tau_N)^2]\equiv\delta_{ij}\sigma_N(\Omega)$, where $\sigma_0\equiv ne^2\tau_N/m$, and $(2\tau_N)^{-1}=\Gamma_N\equiv\Gamma/(1+c^2)$. In the cuprates, $\sigma(T_c)$ is determined by the inelastic scattering rate $1/\tau_\phi(T_c)\sim T_c\gg 1/\tau_N$, as discussed below.

It is possible to obtain analytical results for $\sigma(T,\Omega)$ in various limiting cases. From earlier studies of unconventional superconductors, it is well known that for sufficiently small impurity concentrations, the temperature dependence of transport coefficients may be quite different in two distinct low temperature regimes, separated by a crossover energy T^* [6]. At the lowest temperatures, $T<T^*\ll T_c$, a so-called "gapless" regime exists, in which all superconducting properties reflect the temperature dependence of their normal state analogs, albeit with reduced coefficients scaling with the residual density of quasiparticle states at $T=0$, $N(0)<N_0$. At somewhat higher temperatures, a "pure regime" obtains, where a self-consistent treatment of the self-energies is not required, and transport coefficients generally follow power laws in temperature reflecting the detailed structure of the order parameter nodes. The crossover temperature between these two regimes, T^*, is found to scale as $(\Gamma\Delta_0)^{1/2}$ to within logarithmic factors in the resonant scattering limit $c\simeq 0$.

In the Born limit $c\gg 1$, the crossover temperature varies as $T^*\simeq\Delta_0\exp[-(\Delta_0/\Gamma_N)]$. Thus unless defect concentrations are so high that T_c's are substantially suppressed, the physics of the gapless state is effectively unobservable in the Born limit. The $T\to 0$, $\Omega\to 0$ value of the conductivity is attained only at exponentially small temperatures, and the effective limiting value at experimentally accessible temperatures is the much larger impurity Drude conductivity σ_0, or an appreciable fraction thereof [13,16]. Note that because of the large inelastic scattering rate $1/\tau_\phi(T_c)\sim T_c$, the quantity σ_0 is much larger than the actual normal state conductivity $\sigma(T_c)$ for clean samples. As experiments [7,8,10,17] clearly show a strongly temperature-dependent conductivity at low temperatures, we focus here primarily on the strong-to-intermediate scattering limit, $c\lesssim 1$.

Gapless regime.— In the limit $\Omega\ll T$, we may put $[\tanh\beta\omega/2-\tanh\beta(\omega-\Omega)/2]/2\Omega\to -\partial f/\partial\omega$ in Eq. (2), and require therefore only a small-ω expansion of the kernel $S(\omega,\Omega)$. We focus first on the physics of the gapless regime, in the unitarity limit $c\simeq 0$. In this case the renormalized frequency may be expanded for $\omega\lesssim T^*$, $\tilde{\omega}\simeq i(\gamma+b\omega^2)+a\omega$, where γ, a, and b are constants. For concreteness, we study the $d_{x^2-y^2}$ case, where the constant γ satisfies the self-consistency relation $(\gamma/\Delta_0)^2=(\pi\Gamma)/[2\Delta_0\ln(4\Delta_0/\gamma)]$ for $\Gamma\ll\Delta_0$. It is therefore clear that both γ (or T^*) and the residual density of states $N(0)$ vary as $(\Gamma\Delta_0)^{1/2}$ up to a logarithmic correction. The constants a and b are easily found to be $\frac{1}{2}$ and $1/8\gamma$, respectively. These estimates enable an immediate evaluation of Eqs. (2) and (3) in the gapless regime, yielding $\sigma_{xx}\simeq\sigma_{00}[1+(\pi^2/12)(T/\gamma)^2]$, where $\sigma_{00}=ne^2/m\pi\Delta_0(0)$

for a $d_{x^2-y^2}$ state. Note that if $1/\tau(T_c) \simeq T_c$, and $2\Delta_0/T_c \simeq 6$, then $\sigma_{00} \simeq (1/3\pi)\sigma(T_c)$. In Fig. 1 we present the results of a full numerical evaluation of Eqs. (2) and (3) at $\Omega=0$.

Pure regime.— If, as the authors of Refs. [7,8] believe, their samples are of sufficiently high quality that they observe a linear-T power law for $\delta\lambda(T)$ characteristic of a clean d-wave superconductor in their experiments [1], this would imply that the crossover temperature T^* for these samples is quite low, perhaps $\lesssim 5$ K, the minimum temperature at which data were taken. Thus the "pure" limit is presumably also applicable over the entire temperature range, 5 K $\le T \le$ 90 K, for the conductivity data presented in Refs. [7,8]. The pure regime approximation for $\sigma(T,\Omega)$ is obtained by neglecting self-consistency in the calculation of Σ_0, i.e., replacing $\tilde{\omega}=\omega-\Sigma_0(\tilde{\omega})$ by $\omega-\Sigma_0(\omega)$. We find in this limit ($\gamma \ll T \ll T_c$)

$$\sigma_{xx}(\Omega) \simeq \left(\frac{ne^2}{m}\right)\int_{-\infty}^{\infty} d\omega \left(\frac{-\partial f}{\partial\omega}\right)\left|\frac{\omega}{\Delta_0}\right| \mathrm{Im}\left(\frac{1}{\Omega - i/\tau(\omega)}\right), \tag{4}$$

with $\tau^{-1}=-2\,\mathrm{Im}\Sigma_0(\omega)$, valid for all scattering strengths. It may be of relevance to note at this stage that $\sigma_{xx}(T,\Omega)$ follows a clean linear-T power law if $1/\tau$ is arbitrarily taken to be a constant independent of energy as postulated by Bonn *et al.* for their low-temperature data. However, the microscopic theory in the case of unitary scattering is instead found to give $1/\tau=\pi^2\Gamma\Delta_0/2\omega\ln^2(4\Delta_0/\omega)$, yielding for $\Omega \ll \Gamma\Delta_0/T$

$$\sigma_{xx} \simeq \frac{2}{3}\sigma_0\left(\frac{T}{\Delta_0}\right)^2 \ln^2\frac{4\Delta_0}{T}. \tag{5}$$

For the opposite limit $\Omega \gg \Gamma\Delta_0/T$ we obtain

$$\sigma_{xx} \simeq \left(\frac{ne^2}{m}\right)\frac{\pi^2\Gamma}{2\Omega^2}\ln^{-2}\frac{4\Delta_0}{T}. \tag{6}$$

We note that pair correlations for simple impurity scattering in unconventional superconductors *always* give rise to a nontrivial dependence of $1/\tau$ on energy, in the weak as well as the strong scattering limit. Thus there appears to be no microscopic justification for the assumption of a constant relaxation time in the impurity-dominated regime leading to linear-T behavior. We have seen, however, that the *form* $\sigma \sim n_{qp}\tau/[1+(\Omega\tau)^2]$ is remarkably close to being correct, with the exception that one must formally integrate over the relevant energy-dependent quantities.

For the Born limit, $c \gg 1$, the relaxation rate in a $d_{x^2-y^2}$ state is given simply by $1/\tau(\omega) \simeq \Gamma\omega/\Delta_0$ for $\omega \ll \Delta_0$. In this case, for the pure regime with $\Omega\tau \ll 1$, Eq. (4) reduces to $\sigma_{xx} \simeq \sigma_0 \gg \sigma_{00}$. We have included deviations from the unitarity limit in Fig. 1 to illustrate how the low-frequency Born limit is approached. The downward curvature of the dashed curve in Fig. 1 at the lowest temperatures is indicative of the transiton from the pure to the gapless behavior.

Dynamic effects in impurity-dominated regime.— In the crossover regime between the hydrodynamic ($\Omega\tau \ll 1$) and collisionless ($\Omega\tau \gg 1$) limits, we obtain results for the conductivity which interpolate between Eqs. (5) and (6), and which can display a nearly linear-T behavior over a substantial range. We illustrate this crossover in Fig. 2, and note in particular the transition from the T^2 behavior at low frequencies to the quasilinear behavior for $\Omega/\Gamma \simeq 1$.

Inelastic scattering.— It is possible to try to fit the conductivity data over the entire range using a model self-energy representing the sum of impurity scattering and an inelastic scattering rate term due to spin fluctuations

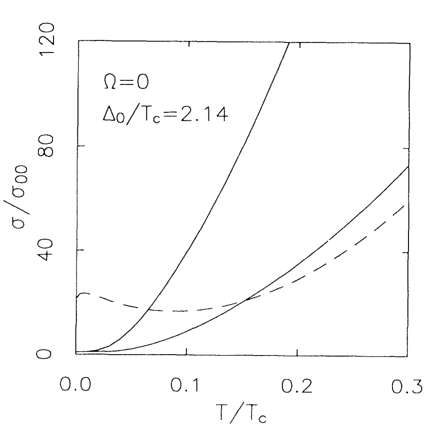

FIG. 1. Normalized conductivity σ/σ_{00} vs reduced temperature T/T_c for a $d_{x^2-y^2}$ state for $\Omega=0$. Upper solid curve, $\Gamma=0.01$, $c=0$; lower solid curve, $\Gamma=0.05$, $c=0$; dashed curve, $\Gamma=0.01$, $c=0.3$.

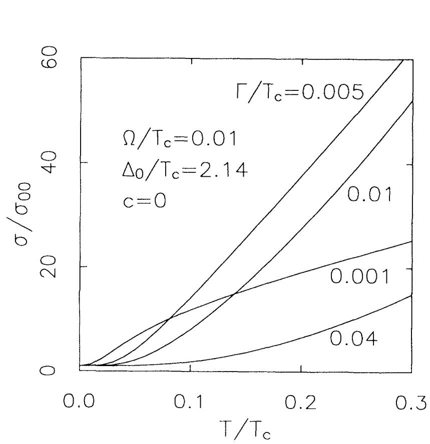

FIG. 2. Normalized conductivity σ/σ_{00} vs reduced temperature T/T_c in the unitarity limit, $c=0$, for $\Omega=0.01T_c$.

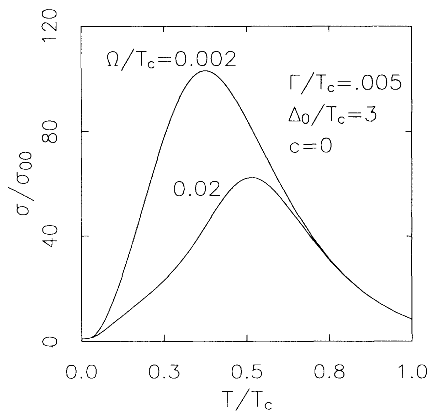

FIG. 3. Normalized conductivity σ/σ_{00} vs reduced temperature T/T_c with $\Gamma=0.005$ and $c=0$, including inelastic scattering according to Ref. [18]. Upper curve, $\Omega=0.002T_c$; lower curve, $\Omega=0.02T_c$.

[18]. While this procedure neglects the real part of the spin fluctuation self-energy and possible vertex corrections, it provides an indication of the relevant energy scales and the role of the inelastic scattering mechanism. Here we adopt for the "inelastic part" of the self-energy a term incorporating the electronic scattering rate due to exchange of antiferromagnetic spin fluctuations as described in Ref. [19]. Such theories have been used to describe the normal state NMR properties of the high-T_c cuprates [20], and in the superconducting state predict nuclear spin-lattice relaxation rates and inelastic scattering rates varying as $(T/T_c)^3$ at low temperatures [21]. In Fig. 3 we plot the calculated conductivity for two frequencies chosen to correspond to the 34.8 and 3.6 GHz measurements of Bonn *et al.* [7,8]. We have chosen the impurity scattering parameters $\Gamma=0.005$, $c=0$ to best fix the positions of the peaks for the two frequencies, as well as the peak heights relative to the normal state conductivity values. It is interesting to note that this choice places the higher-frequency result in the dynamical crossover regime where the low-temperature conductivity is nearly linear in temperature.

In conclusion, we have presented results of a simple theory of losses in a *d*-wave superconductor. In the clean limit $T^* \ll T \ll T_c$, we found that the conductivity has a Drude-like form in which an averaged energy-dependent lifetime enters. For microwave frequencies small compared to the average relaxation rate, σ varies as T^2 at low temperatures, approaching a residual value of order $ne^2/\pi\Delta_0 m$ as $T\rightarrow 0$ [21]. At higher microwave frequencies $\Omega\tau \simeq 1$, a nearly linear T dependence may result, but the lower frequency T^2 dependence of the conductivity differs from the linear T dependence reported. It is not clear at this writing to what extent the large residual losses reported in Refs. [8,9] may be considered intrinsic to the sample; final conclusions regarding the applicability of our model must be reversed until these questions are clarified.

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